TONBRIDGE SCHOOL

Scholarship Examination 2016

MATHEMATICS II

Wednesday 27th April 2016
2.00 p.m.

Time allowed: 1 hour 30 minutes

There are seven questions in this paper
Answer as many questions as you can.
All the questions carry equal marks.
You may attempt the questions in any order.

All answers must be supported by adequate explanation.
Calculators may be used in any question.

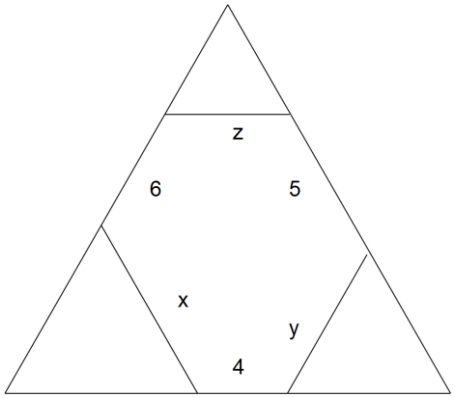
1. A solid cone with base radius r and height h has total surface area S given by the formula $S = \pi r^2 + \pi r\sqrt{r^2 + h^2}$.

(a) If $r = 5$ and $h = 7$, find S.

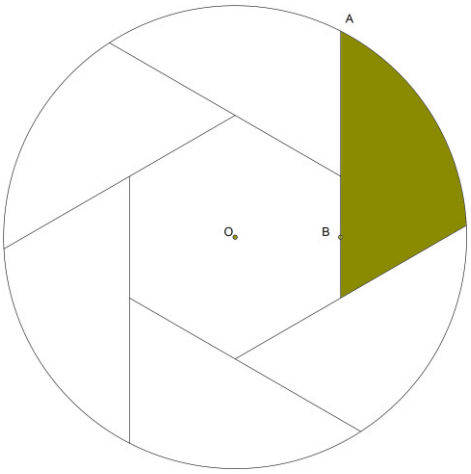
(b) If $S = 200$ and $r = 4$, find h.

(c) If $S = 250$ and $h = 2r$, find r and h.

2. The figure below shows a big equilateral triangle of side-length 12 cm made up of three different small equilateral triangles together with an irregular hexagon with side-lengths (in cm) as shown. Making your method clear, find the values of x, y, z: they are not necessarily whole numbers.



3. The figure shows a regular hexagon of side-length 6 cm with centre O coinciding with the centre of the circle. The radius of the circle is 14 cm. The sides of the hexagon are extended to form six regions identical to the shaded region.



(a) By first finding the area of the hexagon, find the area of the shaded region.

(b) Find the perimeter of the shaded region.
[Hint: Use the right-angled triangle OBA to find AB and then think carefully about the lengths of the straight sides.]

4. In this question, take the radius of the Earth to be 6400 km. You will also need to use the formulae $\frac{4}{3}\pi r^3$ and $4\pi r^2$ for the volume and surface area (respectively) of a sphere of radius r.

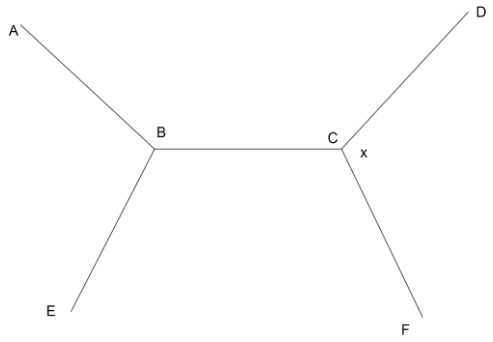
(a) If 70% of the surface of the Earth is covered by oceans, what is the surface area of the oceans?

(b) If the average depth of water in the oceans is 3.8 km, what is the volume of water in the oceans?

(c) If the volume of water in (b) was a sphere, what would its radius be?

(d) What percentage of the Earth's total volume is oceans?

5. In the diagram below, $ABCD$ are four successive sides of a regular polygon with m sides and $EBCF$ are four successive vertices of a regular polygon with n sides. Angle $D\hat{C}F$ is x degrees.



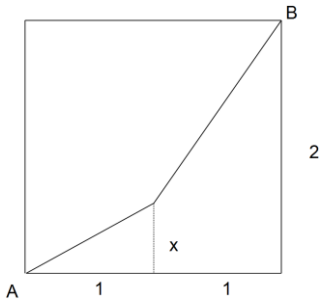
(a) If $m = 20$ and $n = 5$, show that $x = 90°$.

(b) If $m = n$ and $x = 90°$, what is the value of m?

(c) Find another pair of different whole numbers m and n (other than those in (a)) for which $x = 90°$.

TURN OVER

6. In the figure, the length y of the dog-leg path across the square of side-length 2 from A to B is given by the formula $y=\sqrt{1+x^2}+\sqrt{5-4x+x^2}$ where x is the distance shown. (For some values of x the path will go outside the square.)



(a) Make a table of y values (correct to 2 decimal places) corresponding to x values of 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4. Why are the y values for 0.5 and 1.5 the same?

(b) Using your values in (a) and choosing a sensible scale, plot a graph of y against x.

(c) Read-off from your graph the shortest distance from A to B.

(d) For what value of x is the y-value twice the shortest distance from A to B?

7. A sequence of sums involving multiplying fractions goes as follows:

- Sum 1: $1-\frac{1}{2^2}$
- Sum 2: $\left(1-\frac{1}{2^2}\right)\times\left(1-\frac{1}{3^2}\right)\times\left(1-\frac{1}{4^2}\right)$
- Sum 3: $\left(1-\frac{1}{2^2}\right)\times\left(1-\frac{1}{3^2}\right)\times\left(1-\frac{1}{4^2}\right)\times\left(1-\frac{1}{5^2}\right)\times\left(1-\frac{1}{6^2}\right)$

(a) Work out the values of Sum 1, Sum 2, Sum 3 and Sum 4 (not shown), giving your answers as fractions simplified as far as possible.

(b) If the answer to Sum n is $\frac{1009}{2016}$, what is n?

(c) By spotting a pattern in your answers to (a), write down a formula in terms of n for the answer to Sum n.

(d) If the middle fraction in the sequence of fractions for Sum n is $\frac{288}{289}$, what is the answer to Sum n?

(e) Find another value of n (other than $n=4$) for which the answer to Sum n has square numbers in both its numerator and denominator.

END OF PAPER